Interbank network and bank bailouts: Insurance mechanism for non-insured creditors?☆

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Abstract

This paper presents a theory that explains why it is beneficial for banks to engage in circular lending activities on the interbank market. Using a simple network structure, it shows that if there is a non-zero bailout probability, banks can significantly increase the expected repayment of uninsured creditors by entering into cyclical liabilities on the interbank market before investing in loan portfolios. Therefore, banks are better able to attract funds from uninsured creditors. Our results show that implicit government guarantees incentivize banks to have large interbank exposures, to be highly interconnected, and to invest in highly correlated, risky portfolios. This can serve as an explanation for the observed high interconnectedness between banks and their investment behavior in the run-up to the subprime mortgage crisis.

Keywords: bailout, cycle flows, cyclical liabilities, interbank network, leverage
JEL: G01, G21, G28, L14

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1. Introduction

The 2008-2009 financial crisis has prompted many questions about the resilience of the interbank market. Strong growth in the size and density of the interbank network has made concerns such as "too big to fail" and "too interconnected to fail" widespread.[1] However, there is only scarce knowledge of why banks enter into such a high degree of connectivity in the first place, especially since these connections often include cyclical liabilities that could potentially be netted out.

The goal of the present paper is to fill this gap in the literature. We develop a model to show that it can be beneficial for banks to be highly interconnected and even to enter into cyclical liabilities. We claim that this interbank network serves as an insurance mechanism for a bank's creditors if they are not already covered by a deposit insurance (such as, e.g., the FDIC). If a bank failure occurs and there is a non-zero probability that banks will be bailed out by the government, then connections to other banks (e.g., exposures arising from credit default swap (CDS) contracts, bonds, and interbank lending), particularly cyclical liabilities, can actually increase the expected repayment of uninsured creditors. This can be best understood by considering the option pricing approach to explicit and implicit loan guarantees. Merton (1977) shows that the value of these guarantees is akin to a put option. In the case of a bank failure such circular lending activities increase the benefit from this option. This incentivizes banks to be highly interconnected, which implies that many cyclical liabilities occur.[2] We also show that, due to the high interconnectedness, banks are incentivized to invest in correlated assets, thereby increasing the likelihood of a joint default. Banks' risk-shifting incentives increase with their interbank exposure and interconnectedness as well. Therefore, our model helps explain why banks invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis.

Due to the high interconnectedness and resulting cycle flows (i.e. cyclical liabilities), banks are lending to and borrowing from each other large amounts leading to an increased leverage of each bank, without necessarily altering the aggregate relationship between the banking sector and the ultimate creditors or depositors (Shin (2009); Adrian and Shin (2011)), and high systemic risk. However, systemic risk is not only arising from the interconnectedness of banks but can also result from a "joint failure risk arising from the correlation of returns on the asset side of bank balance sheets" (Acharya (2009, p. 225)). We show that the mechanism presented in this paper provides an incentive for banks to increase both types of systemic risk. Moreover, we investigate the interaction between these two sources of systemic risk and show that they cannot be considered individually.

The rest of the paper is organized as follows. Section 2 provides an overview of the related literature. Using a simple example, Section 3 presents our main argument, that due to a positive bailout probability, cyclical liabilities lead to higher expected repayments for uninsured creditors. Section 4 develops our main model and provides implications for

[1] See Minoiu and Reyes (2011), who explore the properties of the global banking network during 1978-2010 and assess its dynamics during financial crises.

[2] Among others, Takács (1988) proves that the expected number of cycle flows increases with the density of the network

the investment behavior of banks. Section 5 shows that interbank connections can lead to risk shifting. Section 6 provides two extensions to our main model. First, we extend our model to a three-region economy and compare different network structures. Second, we introduce risk aversion and show that our main results are not affected. Section 7 concludes.

2. Related Literature

Several empirical papers find that the global banking network has a very high density and a high degree of concentration. Using locational statistics from the Bank for International Settlements on exchange-rate adjusted changes in cross-border bank claims, Minoiu and Reyes (2011) analyze the global banking network and find that, besides a high network density, there exists a positive correlation between network density and the circularity of liabilities (measured by the network's clustering coefficient). Kubelec and Sá (2010) use a cross-country panel dataset of 18 countries to investigate the development of the global financial network over time. They show that the interconnectivity of the global financial network has increased significantly over the past two decades. In line with our results, they find that the global financial network is characterized by a large number of small links and a small number of large links and that the network has become more clustered.

Using micro-level data from Loan Analytics, Hale (2011) shows that in the years 2002-2006 (i.e., before the crisis) the global banking network was characterized by an increasing number of banks, an increasing number of connections between banks, and an increasing number of countries in which banks participate in the global banking network. Moreover, the author finds that this network expansion was mainly driven by a higher interconnectedness of existing banks rather than the entrance of new banks into the global network. This supports our idea that banks were highly connected across countries in the run-up to the financial crisis. Similar evidence can be found for national interbank markets (Wells (2004); Mueller (2006); May et al. (2008)).

Furthermore, there is also a very high interconnectedness in other interbank markets besides the traditional interbank lending market. For example, BIS 2011 shows that banks also have very high cross-exposures due to derivative contracts (mainly CDSs), since banks that sell CDSs in turn also purchase them to hedge their risk. This reduces their net exposure but increases the amount of cyclical liabilities substantially. The extent of these cyclical liabilities can easily be seen from exposure data provided by the International Swaps and Derivatives Association.[3] Comparing the net and gross notional amounts of outstanding CDSs on European sovereign debt shows that the gross is often more than 10 times larger than the net amount. These hedging activities, which in turn entail enormous levels of gross exposure, build up huge counterparty risks. Hence, as the default of AIG demonstrates, as soon as the chain of bilateral netting breaks down, gross exposure becomes net exposure.

[3]See `http://www.isdacdsmarketplace.com/exposures_and_activity/top_10_cds_positions`

Our paper is also related to several strands of the theoretical literature. First, it adds to the literature on liquidity and interbank markets. Pioneering work in this area has been accomplished by Bhattacharya and Gale (1987), who show that banks can co-insure each other through an interbank market against liquidity shocks as long as these shocks are not perfectly correlated. This theme has been taken on by many other papers. For example, Freixas and Holthausen (2005) analyze the scope for international interbank market integration when cross-border information about banks is less precise than home country information. Here, banks can cope with these shocks by investing in a storage technology or can use the interbank market to channel liquidity. Allen et al. (2009) show that the interbank market is characterized by excessive price volatility if there is a lack of opportunities for banks to hedge aggregate and idiosyncratic liquidity shocks. A recent paper by Castiglionesi et al. (2011) shows that there exists a negative relation between a bank's activity in the interbank market and its bank capital because it is optimal for banks to postpone payouts to investors when they are hit by liquidity shocks that cannot be co-insured in the interbank market, in which case interbank activity is low.

In addition, our paper is related to the literature on financial contagion. In Section 6.2 we incorporate our modeling idea into a model setup originally proposed by Allen and Gale (2000). This framework is used by many papers (e.g. Brusco and Castiglionesi (2007), Leitner (2005) and Freixas et al. (2000)). Therefore, we show that the results we find in our main model under the assumption of risk neutrality remain valid when incorporated into a setup of the type proposed by Allen and Gale (2000) and Brusco and Castiglionesi (2007). Similar to these papers, we see the interbank market as an insurance mechanism. In these previous studies, the interbank market is supposed to insure banks against liquidity shocks that result from depositors already withdrawing their money in an intermediate period. In our setting an additional insurance mechanism results from the fact that if a bank is connected to other banks, the expected repayment to uninsured creditors increases in case the bank defaults. This is because even if this specific bank is not bailed out, there nevertheless exists a positive probability that the next bank in the chain will be. If markets have reached a high network density with high capital flows, implying that many and large cycle flows exist, then ultimately the failing bank will receive funds from banks it is connected to if they are bailed out.

Similar to our model, Castiglionesi and Navaro (2010) use a banking network with core and periphery banks (uninsured creditors) that differ with respect to their interconnectedness (and investment risk) and establish conditions under which fragility is an optimal feature of financial networks. Cukierman and Izhakian (2011) develop a micro-founded general equilibrium model of the financial system composed of ultimate borrowers, ultimate lenders, and financial intermediaries and investigate the impact of bailout uncertainty on leverage, interest rates, the volume of defaults, and the real economy. Our approach differs from theirs in that we start by assuming a fixed bailout probability and investigate how it affects the expected repayment uninsured creditors receive from a bank (and hence the interest rate the bank is able to pay uninsured creditors) under different network structures. David and Lehar (2011) also present a mechanism that incentivizes banks to create cyclical liabilities. In the case where banks have perfect information about the interbank network and the liabilities of all banks, cyclical liabilities can act as a com-

mitment device to facilitate mutual private sector bailouts. In contrast, we investigate the effect of possible government bailouts on the incentives of banks to create such liabilities.

Lastly, our paper relates to the literature on bank bailouts. Acharya and Yorulmazer (2007) focus on whether governments have an incentive to bail out banks ex post if they engaged in herding behavior ex ante. Diamond and Rajan (2002) show that bailouts alter available liquidity in the economy and distinguish between well-targeted bailouts (which can be beneficial) and poorly targeted ones that can lead to a systemic crisis. Gorton and Huang (2004) argue that there is a potential role for governments to provide liquidity through, for example, bank bailouts to reduce the problem of agents hoarding liquidity inefficiently.

3. Main Idea

To illustrate our main idea, we use a very simple framework similar to that of Rotemberg (2011). We assume that the interbank market consists of a few core banks and some uninsured creditors (e.g., mutual funds, bondholders, regional banks). One of the core banks has an investment project that costs one unit in the first period and generates a return $R > 1$ in the second period with probability λ and a return of zero otherwise. The only source of capital to fund this project is to borrow from the uninsured creditors. In return for the initial funding, the bank must repay R_D to its uninsured creditor. All parties are risk-neutral.

We develop the intuition of our model in two steps. First, we discuss a situation without network connections to other core banks. At $t = 0$ the bank (B_A) borrows one unit from the uninsured creditor (C) and invests in a project (P). In the second period, the cash flow from the project is realized. If the project is successful, the bank receives an amount R and is able to fully repay its uninsured creditor. If the project fails and the bank is not bailed out, the uninsured creditor receives no repayment. Conversely, if the government bails out the bank (i.e., takes over the bank and settles all its liabilities), the creditor again receives his full repayment (see Figure 1).

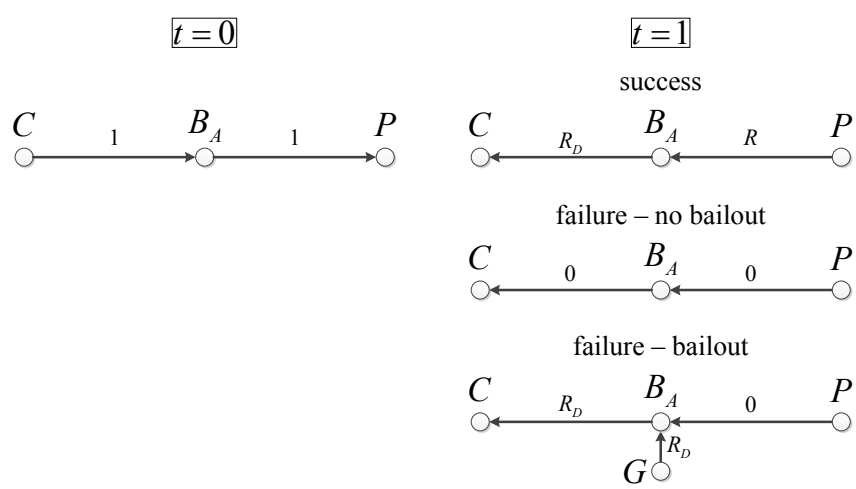

Figure 1: Capital flows without interbank market and zero bailout probability

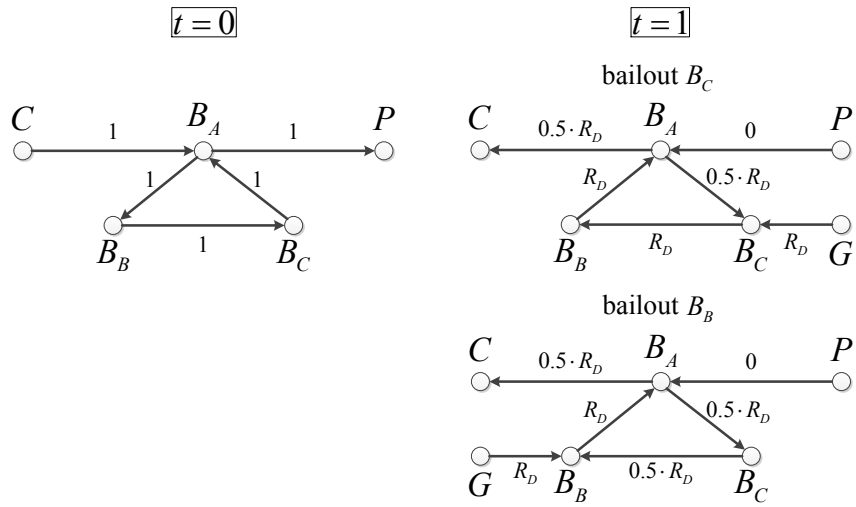

Figure 2: Capital flows with interbank market and bailout

In a second step we allow the bank to establish an interbank network at $t = 0$ by lending one unit of capital that, for example B_A receives from its uninsured creditor in a circular way. To be precise, bank B_A lends one unit of capital to bank B_B, which in turn lends it to bank B_C, from which the capital flows back to B_A and is then invested into the project. For now, we assume that banks B_B and B_C do not have any other investments. We relax this assumption in the next sections. Moreover, for ease of illustration, we assume that the gross interest rate on the interbank market is R_D as well. If the project is successful, B_A receives the project return R and uses it to settle its liabilities with B_C.[4] After receiving the payment from B_B it repays its uninsured creditor. If the project fails, bank B_A defaults since it cannot repay its creditors. If the government steps in and bails out bank B_A, both the uninsured creditor of B_A and bank B_C receive their full repayment R_D, implying that all claims are settled in this case. If the government refuses to bail out B_A, B_C defaults as well. Now the government (not necessarily the same one as in the case of B_A, since B_C may be established in another country) must decide whether to bail out B_C. If it does, it takes over B_C and settles its liabilities. Therefore B_B receives R_D from B_C and hence B_B can pay back its debt to B_A. However, B_A has total liabilities of $2R_D$ and is therefore still unable to meet all its obligations. Consequently, the funds B_A received from B_B must be divided among the creditors of B_A, that is, the uninsured creditor of B_A, on the one hand, and B_C, on the other hand.

The common procedure in bankruptcy proceedings is for debt to be paid back on a pro rata basis once a default occurs. Therefore, each creditor receives $\frac{1}{2}R_D$. Since the

[4]Throughout the paper we assume that, as soon as there exists a clearing payment vector, the banks use this vector to settle all liabilities in the network. This no longer holds if the sequence of payments is chosen in a less sophisticated manner. In this case, banks can still default, even though there is enough liquidity in the system to settle all claims. However, this would only reinforce our mechanism, since it would increase the value of the government's implicit guarantee.

government takes over B_C, it receives this amount. However, it has to pay R_D to bail out the bank and hence records a loss of $\frac{1}{2}R_D$. The case where B_C is not bailed out but B_B is can be described analogously. The corresponding cash flows (in case the project fails and one of the other banks is bailed out) are presented in Figure 2. Hence, in case there is a positive probability of a government bailout if a bank defaults, the bank can considerably increase the expected repayment of its uninsured creditor by first channeling funds through the interbank market and only lending them out to the ultimate borrower afterwards. This is because the uninsured creditor receives a positive repayment as soon as at least one of the banks is bailed out.

If the bank has the bargaining power, creditors will demand a lower interest rate (risk premium) given the existence of an interbank network (the participation constraint of uninsured creditors is already binding for lower values of R_D) which considerably reduces the bank's borrowing cost. This in turn leads to higher profits for the bank, which can help explain the comparatively high return-on-equity ratios of banks. If, on the other hand, the uninsured creditor has the bargaining power, he will increase his expected repayment by increasing R_D until the participation constraint of the owners of the bank is just binding. Furthermore, creditors will only deposit money in banks that are part of an interbank network, since the expected repayment in this case is higher than when the bank is not connected to others via an interbank market.

Note that the described mechanism can be reinforced by channeling more than one unit of capital through the interbank market. For example, this can be realized by repeating the circular lending procedure a couple of times (e.g., K repetitions lead to an interbank network exposure of K). This increases the expected repayment to the uninsured creditor even further. Moreover, it is easy to see that the expected repayment to the uninsured creditor can also be increased by increasing the number of banks in the interbank network.

4. The Main Model

Having described the main mechanism we now formalize our idea and develop our main model. We consider an economy that consists of two dates $t = 0$ and $t = 1$ and two different regions, A and B (which can be interpreted as, e.g., two different countries). Each region is comprised of a continuum of identical banks. We assume that, due to competition, all banks adopt the same behavior and can thus be described by a representative bank (protected by limited liability). The representative bank in region A (B) is denoted by B_A (B_B). In line with Allen and Gale (2000), these banks can establish an interbank market (network) by exchanging an arbitrary amount of interbank deposits K at $t = 0$ in return for a payment of KR_D at $t = 1$. This is a simplified approach to model the cycle flows that otherwise result from a high degree of market density.[5]

Furthermore, we assume that there exists an uninsured creditor (endowed with c units

[5]Note, however, that there exists some anecdotal evidence from German Landesbanks that even this kind of bilateral circular lending exists on the interbank market. For example, a 2006 report by Fitch describes that after the abolition of the explicit state guarantee, Landesbanks bought bonds from each other in large amounts, thereby creating "cyclical liabilities" bilaterally.

of capital at date $t = 0$) and one investor who provides equity financing to the bank in each region. Creditors are denoted C_A and C_B in regions A and B, respectively. This contract takes the form of a standard debt contract; that is, it cannot be made contingent on either the realization of the investment or the realization of the state of nature. Lastly, we consider a government in each region. All actors are risk neutral.

We consider a situation where each bank has access to two investment possibilities in two different industries (denoted 1 and 2), as in Acharya and Yorulmazer (2007). Both investments need an initial amount of capital o which is normalized to one. One can think of these investment opportunities as portfolios of loans to firms in one of the two industries. More precisely, bank B_A (B_B) can lend to firms in industry A_1 or A_2 (B_1 and B_2). If in equilibrium banks decide to lend to firms in the same industry, that is, they either lend to A_1 and B_1 or to A_2 and B_2, then the returns of their loan portfolios are assumed to be perfectly correlated ($\rho = 1$). However, if they decide to invest in different industries, we assume that the returns are uncorrelated ($\rho = 0$).

The investment opportunities are only available at date $t = 0$. Both portfolios generate a return of R with probability λ or a return of zero with probability $(1 - \lambda)$ at $t = 1$. Note that we assume that the investment opportunity has a positive net present value (NPV), that is, $\lambda R > 1$, and that $\lambda \geq 1/2$. The latter can be motivated by considering the Value at Risk constraint of the Basel Accord, which states that banks must choose a minimum quality for their loan portfolio to limit their default probability. Consequently, the decision in which industry to invest only affects the correlation of returns, but not their magnitude. This structure allows us to determine whether interbank connections incentivize banks to invest in correlated investments.

Finally, to model risk-neutral investors we follow Allen and Gale (2005) and Brusco and Castiglionesi (2007) in that we assume that the equity investor I_A (I_B) in region A (B) is endowed with e units of capital at $t = 0$ and has no endowment at date $t = 1$. He can use his endowment for either consumption or to buy bank shares. In the latter case the investor is entitled to receive dividends at $t = 1$ (denoted by d_1). His utility is then given by

$$u(d_0, d_1) = d_0 \lambda R + d_1$$

Since an investor can obtain a utility of $e\lambda R$ by immediately consuming his initial endowment (consumption at $t = 0$ is denoted by d_0), he has to earn an expected return of at least λR on his invested capital in order to give up consumption at date $t = 0$. By investing an amount e_0 at $t = 0$, the equity investor obtains a lifetime utility of $(e - e_0)\lambda R + d_1$. Hence, he will only buy bank shares if the expected utility from doing so is higher than the utility he would get from immediately consuming his endowment, that is, if $(e - e_0)\lambda R + E[d_1] \geq e\lambda R$. This leads to the following participation constraint for investors:

$$E[d_1] \geq \lambda e_0 R$$

Under the assumption of perfect competition in the banking market (i.e., creditors have all the bargaining power), this constraint will be binding. Hence, the total amount of funds provided to the bank is given by $c + e_0 = o = 1$. Due to the prevailing capital structure of banks, we assume that $c > e_0$, that is, that the bank has more debt than

equity. From now on we will suppress the time index. The timing of our model is as follows:

$t = 0$	$t = 1$
Investors provide equity e_o	Cash flows are realized
Banks exchange K	Governments decide on bailouts
Banks raise debt capital c	
Banks invest in loan portfolio	

If both investments are successful, the banks are able to settle their interbank claims, repay the uninsured creditors, and pay the investors a positive dividend. If, however, the investment of one or both banks fails, either one or both banks may not be able to meet their liabilities and will consequently default. In case of a default we assume that there is a positive probability α that the government of the respective country will step in and bail out the bank, that is, take over the bank and repay all its liabilities.[6] It would be reasonable to assume that α is initially increasing in the interconnectedness of the bank (too interconnected to fail), its balance sheet size (too big to fail) and the number of failing banks (too many to fail). However, as soon as the bank reaches a critical size, it becomes "too big to save" and therefore its bailout becomes impossible and α drops to zero. Since we want to isolate the direct effect that cycle flows have on the expected repayment of uninsured creditors, we assume that the bailout probability is not increasing in either the balance sheet size of the bank or its interconnectedness or the number of failing banks. Making the bailout probability increasing with one of these factors would reinforce our results, since this gives banks an incentive to increase their interconnectedness even further. However, we capture the argument of being too big to save by assuming that the bailout probability becomes zero as soon as a bank's balance sheet exceeds a critical threshold $\overline{L} >> R$.[7] If the bank's size reaches this threshold, the government will no longer be able to provide enough capital to bail it out. Therefore, α becomes:

$$\alpha = \begin{cases} \alpha_B & \text{if } (c+K)R_D \leq \overline{L} \\ 0 & \text{if } (c+K)R_D > \overline{L} \end{cases}$$

Consequently, the payments to the uninsured creditors and investors depend on the performance of the loan portfolio and on whether a bank is bailed out if a default occurs. As described in the previous section, we can derive our results no matter which

[6] This is a simplification, since the bailout probability for different banks is probably correlated. However, for our mechanism to work, it is sufficient that the bailout probabilities are not perfectly correlated. This is certainly true if the banks are established in different countries. Furthermore, during the recent crisis, the bailout of Bear Stearns, and the default of Lehman Brothers show that bailout decisions are also not perfectly correlated within the same country.

[7] This assumption is supported by the findings of Acharya et al. (2011). These authors show that financial sector bailouts and sovereign credit risk are linked. On the day of the announcement of large bailouts, the CDS spreads on government bonds rose significantly. If a government has to spend very high amounts to rescue a bank, it becomes virtually impossible to obtain funding for this bailout at acceptable terms. Thus, once a bank is too large, it can no longer be rescued.

party (i.e., creditors or banks) has the bargaining power. To ensure consistency with our extension that considers risk-averse creditors, we assume here that the creditors have all the bargaining power. Due to perfect competition in the banking sector, this implies that banks seek to maximize the repayment of uninsured creditors by choosing the parameters R_D, ρ, and K. Having described the setup, we now return to our main questions in this section: Which level of interconnectedness do banks choose and do they prefer to invest in correlated assets?

Both aspects are important to consider, since they both increase systemic risk. On the one hand, interconnectedness leads to systemic risk resulting from spillover effects that are transmitted through the interbank market (even without correlation on the asset side of the banks' balance sheet). On the other hand, even without being interconnected, correlation increases systemic risk due to possible joined bank failures. The following analysis investigates the interaction between these two sources of systemic risk and determines how interconnectedness influences the banks' investment decision, that is, whether they invest in correlated loan portfolios. To analyze this issue we derive the highest expected repayment banks can achieve with an investment correlation of zero and one, respectively. Then we compare the resulting repayments to determine which of the two yields a higher return for uninsured creditors.

4.1. Positively Correlated Investments

Consider first the situation where bank investments are perfectly positively correlated, that is, $\rho = 1$. In this case there are five different outcomes (depending on the success of the investments and whether the banks are bailed out or not), depicted in Table 1.

$\rho = 1$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	I_A	I_B
S_1	λ	S	S	N	N	cR_D	cR_D	$R - cR_D$	$R - cR_D$
S_2	$(1-\lambda)\alpha^2$	F	F	B	B	cR_D	cR_D	0	0
S_3	$(1-\lambda)(1-\alpha)\alpha$	F	F	B	N	cR_D	$cR_D \frac{K}{c+K}$	0	0
S_4	$(1-\lambda)(1-\alpha)\alpha$	F	F	N	B	$cR_D \frac{K}{c+K}$	cR_D	0	0
S_5	$(1-\lambda)(1-\alpha)^2$	F	F	N	N	0	0	0	0

Table 1: Capital flows for investment correlation of $\rho = 1$

Column 1 presents the five different states, while column 2 presents the probability of each given state occurring. Columns L_A and L_B show whether the investments of banks B_A and B_B are successful (S) or not (F). Columns B_A and B_B show whether banks B_A and B_B are bailed out by the government (B) or not (N). The columns C_A and C_B show the repayment of uninsured creditors, while columns I_A and I_B show the dividends the equity holders receive. To understand the cash flows presented in Table 1, first note that if either both investments are successful (S_1) or both banks are bailed out (S_2), the uninsured creditors of both banks will receive their full repayment. These states only differ with respect to the dividend paid to the investor, since in the case of a bailout the government takes over the bank and thus has the residual claim. If only one bank is bailed out (S_3 and S_4), then the creditor of this bank will receive his full repayment whereas the creditor of the other bank will receive only a fraction $\frac{K}{c+K}$ of his claim cR_D. Since the model is symmetric, it is sufficient to focus on the optimization problem of one of the banks. Hence, we only analyze the behavior of bank B_A. Due to perfect competition,

bank B_A wants to maximize the expected repayment to its uninsured creditor C_A. Thus, its optimization problem becomes:

$$\max_{R_D,K} U_1 = \lambda c R_D + (1-\lambda)\left[\alpha c R_D + \alpha(1-\alpha) c R_D \frac{K}{c+K}\right] \tag{1}$$

subject to

$$E[d_1] \geq \lambda e R$$

The objective function consists of the following parts: With probability λ the investment of the bank is successful and creditors receive their contractually specified repayment cR_D. With probability $(1-\lambda)$ the investment fails. In this case the return of the creditors depends on whether the banks are bailed out or not. Specifically, if bank B_A is bailed out (which happens with probability α), the government repays all liabilities and hence its creditors again receive the full repayment. If, however, the government decides not to bail out bank B_A, the repayment depends on whether bank B_B is bailed out (remember that since investment outcomes are perfectly correlated, bank B_B is in default as well). If bank B_B is not bailed out either, the repayment is clearly zero. However, if bank B_B is bailed out, then the government injects funds of $R_D(c+K)$. This allows bank B_B to settle all its claims. Therefore, B_A receives $R_D K$ and has to split these proceeds on a pro rata basis (it owes money to its uninsured creditor C_A and bank B_B). Therefore, the uninsured creditors of bank B_A will receive a share $\frac{c}{c+K}$ of the funds bank B_A received from B_B. Furthermore, the binding participation constraint of the equity holder implies

$$E[d_1] = e\lambda R \Rightarrow \lambda(R - cR_D) = e\lambda R \Rightarrow R_D = R$$

Inserting $R_D = R$ into (1) yields the following maximization problem:

$$\max_{K} U_1 = \lambda c R + (1-\lambda)\left[\alpha c R + \alpha(1-\alpha) c R \frac{K}{c+K}\right] \tag{2}$$

Since R and c are given, it will depend on K whether the government will be able to fully repay the bank's liabilities in case of a bailout. Let $\overline{K_1}$ denote the interbank exposure where the government is just able to repay all liabilities; this will be given by $\overline{K_1} = \frac{\overline{L}}{R} - c$. In the following we split the amount of interbank deposits into two intervals. For $K \in [0, \overline{K_1}]$ (government will be able to repay all liabilities and $\alpha = \alpha_B$) the first-order condition of the objective function becomes

$$\frac{\partial U_1}{\partial K} = R\frac{\alpha_B(1-\alpha_B)(1-\lambda)c^2}{(c+K)^2} > 0 \tag{3}$$

If, on the other hand, banks increase their exposure to an even higher level, that is, $K \in (\overline{K_1}, \infty]$, then the government will not be able to provide enough funds to settle all the liabilities of the failed bank and the bailout probability α drops to zero. Hence, the expected repayment of C_A drops to $\lambda c R$.

Thus, the expected utility of the uninsured creditors is increasing in K as long as

$R(c+K) < \overline{L}$. This implies that banks will choose an amount of interbank deposits $K = \overline{K_1}$ such that $R(c+K) = \overline{L}$.[8] Increasing cross-exposure on the interbank market beyond this threshold decreases the expected repayment of the uninsured creditor. Therefore, the highest expected utility for the creditor that can be achieved when choosing a correlation $\rho = 1$ is given by

$$\overline{U_1} = \lambda cR + (1-\lambda)\left[\alpha_B cR + \alpha_B(1-\alpha_B)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2}\right] \tag{4}$$

Our findings can be summarized in the following corollary.

Corollary 4.1. *If banks choose perfectly correlated investments (given a positive bailout probability), they will increase their interbank exposure up to the threshold $K = \overline{K_1}$, such that their total liabilities equal $\overline{L}$, that is, to a level that makes it just possible to bail them out in case of default.*

Proof The proof follows from the previous discussion. QED

To understand why it makes sense intuitively to choose such a high level of interbank deposits, one must consider two opposing effects. On the one hand, higher exposure increases the funds injected by the government in case of a bailout and hence increases the funds that can be split among a bank's creditors. On the other hand, a higher amount of interbank deposits decreases the fraction that the uninsured creditor of the bank that is not bailed out receives, since $\frac{c}{c+K}$ decreases in K. Since the first effect outweighs the second effect, banks choose the highest possible liabilities $\overline{L}$.

4.2. Uncorrelated Investments

We next turn to the case where banks decide to invest in different industries, that is, $\rho = 0$. Here, two scenarios must be considered. On the one hand, the interbank exposure can be chosen such that even if the one bank's investment is successful but the other bank's investment fails, the first bank will be unable to repay its obligations and hence financial contagion will occur. On the other hand, if the exposure is low enough, a successful bank will stay solvent no matter what happens to the other bank. Let K^* denote the "switching point", that is, the level of interbank exposure where a successful bank will just stay solvent, even if the other bank fails (see the Appendix for the derivation of K^*). The different possibilities for the cash flows are presented in Tables 2 and 3, where the notation is as described before. It is crucial to note that the interest rate R_D differs between the two possibilities, since the participation constraint of the equity investors differs. Table 2 presents the cash flows for $K \leq K^*$.

[8] Due to minimum capital requirements, banks must often back interbank loans with equity capital. Hence, depending on the risk weights of interbank loans and the banks' amounts of equity, there may be an individual upper limit for the banks' interbank exposure K that prevents banks from increasing their exposure to $K = \overline{K_1}$.

$\rho = 0$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	I_A	I_B
S_1	λ^2	S	S	N	N	cR_D^1	cR_D^1	$R - cR_D^1$	$R - cR_D^1$
S_2	$(1-\lambda)^2\alpha^2$	F	F	B	B	cR_D^1	cR_D^1	0	0
S_3	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	B	N	cR_D^1	$cR_D^1\frac{K}{c+K}$	0	0
S_4	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	N	B	$cR_D^1\frac{K}{c+K}$	cR_D^1	0	0
S_5	$(1-\lambda)^2(1-\alpha)^2$	F	F	N	N	0	0	0	0
S_6	$\lambda(1-\lambda)\alpha$	S	F	N	B	cR_D^1	cR_D^1	$R - cR_D^1$	0
S_7	$\lambda(1-\lambda)\alpha$	F	S	B	N	cR_D^1	cR_D^1	0	$R - cR_D^1$
S_8	$\lambda(1-\lambda)(1-\alpha)$	S	F	N	N	cR_D^1	$cR_D^1\frac{K}{c+K}$	X_0	0
S_9	$\lambda(1-\lambda)(1-\alpha)$	F	S	N	N	$cR_D^1\frac{K}{c+K}$	cR_D^1	0	X_0

Table 2: Outcomes for $K \leq K^*$, where $X_0 = R - cR_D^1 - cR_D^1\frac{K}{c+K}$ - No contagion

States $S_1 - S_5$ parallel the respective outcomes in Table 1. Things differ from the results of Table 1 if only one investment fails, depending on whether the successful bank stays solvent (no contagion; see Table 2) or also becomes insolvent (see Table 3). If the interbank exposure is low enough ($K \leq K^*$) such that there is no contagion, then the successful bank can always fully repay its uninsured creditor, whereas the creditor of the unsuccessful bank will only receive the full amount if this bank is bailed out (S_6 and S_7 in Table 2). Otherwise, he will get just a fraction of his repayment (S_8 and S_9 in Table 2). If, on the other hand, the interbank exposure is higher than the threshold K^*, the successful bank will not be able to settle its interbank liabilities and, on top of that, will be unable to fully repay its creditor. Depending on which bank (if any) is bailed out, the creditors of both the successful and the failed bank receive either their full repayment or just a fraction ($S_6 - S_{11}$ in Table 3).

$\rho = 0$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	I_A	I_B
S_1	λ^2	S	S	N	N	cR_D^2	cR_D^2	$R - cR_D^2$	$R - cR_D^2$
S_2	$(1-\lambda)^2\alpha^2$	F	F	B	B	cR_D^2	cR_D^2	0	0
S_3	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	B	N	cR_D^2	$cR_D^2\frac{K}{c+K}$	0	0
S_4	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	N	B	$cR_D^2\frac{K}{c+K}$	cR_D^2	0	0
S_5	$(1-\lambda)^2(1-\alpha)^2$	F	F	N	N	0	0	0	0
S_6	$\lambda(1-\lambda)\alpha$	S	F	N	B	cR_D^2	cR_D^2	$R - cR_D^2$	0
S_7	$\lambda(1-\lambda)(1-\alpha)\alpha$	S	F	B	N	cR_D^2	$cR_D^2\frac{K}{c+K}$	0	0
S_8	$\lambda(1-\lambda)(1-\alpha)^2$	S	F	N	N	$R\frac{c+K}{c+2K}$	$R\frac{K}{c+2K}$	0	0
S_9	$\lambda(1-\lambda)\alpha$	F	S	B	N	cR_D^2	cR_D^2	0	$R - cR_D^2$
S_{10}	$\lambda(1-\lambda)(1-\alpha)\alpha$	F	S	N	B	$cR_D^2\frac{K}{c+K}$	cR_D^2	0	0
S_{11}	$\lambda(1-\lambda)(1-\alpha)^2$	F	S	N	N	$R\frac{K}{c+2K}$	$R\frac{c+K}{c+2K}$	0	0

Table 3: Outcomes for $K > K^*$ - Contagion

In a next step we must compare the expected repayments of the uninsured creditor in these two scenarios, that is, $K \leq K^*$ and $K > K^*$. To do so, we first derive the precise values of R_D^1 and R_D^2 from the binding participation constraint of the equity holder. If $K \leq K^*$, we obtain from the constraint $E[d_1] \geq e\lambda R$

$$\lambda^2(R - cR_D^1) + \lambda(1-\lambda)\left[\alpha(R - cR_D^1) + (1-\alpha)\left(R - cR_D^1 - cR_D^1\tfrac{K}{c+K}\right)\right] \geq (1-c)\lambda R$$
$$\Rightarrow R_D^1 = R\tfrac{c+K}{c+2K-K[\lambda+(1-\lambda)\alpha]}$$

The interest rate R_D^1 is decreasing in the interbank exposure K since, due to an increased K, a higher fraction of the investment return is paid from bank B_A to B_B and thus creditor

C_B receives a higher repayment. This reduces the dividend payment of I_A. Hence, to satisfy the investor's participation constraint, R_D^1 must be reduced. Furthermore, R_D^1 is increasing in the success probability λ, the bailout probability α, and the investment return R. The success probability λ increases the probability that the equity investor will receive a dividend payment. Hence, a lower dividend payment is sufficient to satisfy his participation constraint. An increase in the bailout probability α makes the bailout of the other bank more likely in case of default. This increases the probability that the investor will receive the full dividend payment $R - cR_D^1$.

Furthermore, the interest rate R_D^1 depends on the amount of debt borrowed from the uninsured creditors. Consider, for example, bank B_B. If this bank increases c, C_B is entitled to a higher fraction of the bank's liquidation value. Hence, the fraction paid back into the interbank market is lower. This reduces the dividend payment equity investor I_A receives and thus the interest rate R_D^1 must be reduced. If, on the other hand, bank B_A increases c, investors have to invest less equity and the interest rate R_D^1 can be increased. Conversely, if $K > K^*$, we obtain

$$\begin{aligned} &\lambda^2(R - cR_D^2) + \lambda(1-\lambda)\alpha(R - cR_D^2) \geq (1-c)\lambda R \\ &\Rightarrow R_D^2 = R\left(\frac{\lambda + (1-\lambda)\alpha - (1-c)}{c[\lambda + (1-\lambda)\alpha]}\right) \end{aligned} \tag{5}$$

Therefore, as soon as $K > K^*$, a change in K does not alter the dividend payment to I_A and hence no longer changes the interest rate R_D^2. For the same reasons as for R_D^1, R_D^2 is increasing in the success probability λ, the bailout probability α, the debt amount c, and the investment return R. Given our assumptions on λ, c, and e, we can make sure that $0 < R_D^2 < R$. Plugging the value of R_D^1 (since we approach K^* from below) into the formula for K^* in equation (33) (see the Appendix) yields

$$K^* = \frac{c(1-c)}{\lambda + (1-\lambda)\alpha - 2(1-c)}$$

Hence, to obtain a positive interbank exposure K for which the successful bank stays solvent (in case one bank is successful and the other is not), it must hold that $\lambda + (1-\lambda)\alpha - 2(1-c) > 0$. Otherwise, we can restrict our analysis to the case $K > K^*$. Therefore, if the investment correlation is zero, the overall utility of the uninsured creditors (depending on the amount of interbank deposits) is

$$\begin{aligned} U_0(K \leq K^*) &= [\lambda + (1-\lambda)\alpha]cR_D^1 + (1-\lambda)(1-\alpha)[\lambda + (1-\lambda)\alpha]cR_D^1\frac{K}{c+K} \\ &= [\lambda + (1-\lambda)\alpha]cR \\ U_0(K > K^*) &= \left[\alpha(1+\lambda) + \lambda^2(1-2\alpha) - \alpha^2\lambda(1-\lambda)\right]cR_D^2 + \lambda(1-\lambda)(1-\alpha)^2R \\ &+ \alpha(1-\lambda)(1-\alpha)cR_D^2\frac{K}{c+K} \end{aligned}$$

We now have to compare the utility of the creditors for the different levels of interbank deposits. In the Appendix, we formally show that banks have an incentive to choose a

level of interbank deposits $\overline{K_0} = \frac{\overline{L}}{R_D^2} - c$ in case $(c + K^*)R_D^1 < \overline{L}$. If, on the other hand, $(c + K^*)R_D^1 \geq \overline{L}$, banks will be indifferent between all possible interbank exposures in the interval $K = [0, \overline{K_0}]$. Hence, if $(c + K^*)R_D^1 < \overline{L}$, the highest expected utility for the non-insured creditor that can be achieved when choosing a correlation of $\rho = 0$ is given by

$$\begin{aligned} \overline{U_0} &= \left[\alpha(1+\lambda) + \lambda^2(1-2\alpha) - \alpha^2\lambda(1-\lambda)\right] cR_D^2 + \lambda(1-\lambda)(1-\alpha)^2 R \\ &+ \alpha(1-\lambda)(1-\alpha)\overline{L}\frac{c\overline{K_0}}{(c+\overline{K_0})^2} \end{aligned}$$

Furthermore, if $(c + K^*)R_D^1 \geq \overline{L}$, the maximal expected utility becomes

$$\overline{U_0} = \lambda cR + (1-\lambda)\alpha cR$$

This finding can be summarized in the following corollary:

Corollary 4.2. *If banks choose uncorrelated investments (given a positive bailout probability), two scenarios must be considered:*

a) *If $(c + K^*)R_D^1 < \overline{L}$, banks will increase their interbank exposure up to the threshold $K = \overline{K_0}$,*

b) *If $(c + K^*)R_D^1 \geq \overline{L}$, banks will be indifferent between all possible interbank exposures in the interval $K = [0, \overline{K_0}]$.*

Proof See the Appendix. QED

Hence, intuitively, two cases must be distinguished. On the one hand, the level of interbank exposure above which contagion occurs may be low enough so that the bank can be bailed out ($(c + K^*)R_D^1 < \overline{L}$). Then it is always optimal to increase the interbank exposure K to a level that just enables the government to bail out the bank ($K = \overline{K_0}$), implying that contagion can occur. This is due to the fact that as soon as the interbank exposure K exceeds the contagion threshold K^*, a change in K no longer alters the interest rate R_D^2. Therefore, the only downside for the bank's creditor in choosing a higher K is due to the states where only his own bank is successful. In such cases a higher interbank exposure implies that a higher fraction of the return generated by that bank is transferred to the creditor of the other bank. However, the creditor benefits in the same way in case his own bank fails while the other bank is successful. The benefits and costs of the respective states add up to zero. An additional upside of a higher K results from the state where both banks fail and the other bank is bailed out (as described in Section 4.1). Taken together, these effects incentivize banks to increase their interbank exposure up to $\overline{K_0}$.

If, on the other hand $(c + K^*)R_D^1 \geq \overline{L}$, the banks are unable to choose an interbank exposure that leads to contagion and at the same time allows the government to bail out the bank ($K \leq \overline{K_0} < K^*$). In this interval for K, equity investors receive a dividend

payment whenever their own bank is successful. This payment, however, only depends on the interbank exposure if the other bank fails and is not bailed out. In this state, if K is increased, R_D^1 must be reduced. Consequently, this effect will reduce the creditor's expected repayment. However, there is also a countervailing effect if the bank increases K. Due to the effect previously described, an increase in K increases the expected repayment to the creditor in cases where his own bank fails but the other bank is either successful or bailed out. These two effects offset each other such that the expected repayment to the uninsured creditor is not influenced by the choice of the interbank exposure in the interval $K \leq \overline{K_0} < K^*$.

4.3. Comparison of Correlated and Uncorrelated Investments

What remains is to show under which correlation structure uninsured creditors receive a higher expected repayment. In the Appendix we formally prove that $\overline{U_1} > \overline{U_0}$ will always hold, implying that banks will always choose investments that are perfectly correlated. This main finding can be summarized in the following proposition.

Proposition 4.3. *If banks are connected via an interbank market and there is a non-zero bailout probability, it is optimal for them to invest in correlated assets. Moreover, they have an incentive to increase their interbank exposure until their total liabilities equal $\overline{L}$, that is, the highest amount that still allows the bank to be bailed out.*

Proof See the Appendix. QED

To understand why this result holds, first note that the contractually specified repayment R_D^2 is lower if investments are uncorrelated (since $R_D^2 < R$). This is because the investor has a lower probability of receiving a dividend payment if investments are independent. Hence, a higher dividend amount (and thus a lower contractually specified repayment for the creditor) must be paid to satisfy the investor's participation constraint. Moreover, the probability of receiving the full repayment is lower as well, if investments are uncorrelated. Furthermore, the lowest positive repayment the creditors can receive is higher with perfectly correlated investments (this lowest repayment occurs with the same probability under either correlation structure). Finally, if investments are uncorrelated, there is an additional "intermediate" state in which the creditor receives only a fraction of his repayment (again as in the case of a zero bailout probability). These three effects together more than offset the fact that a zero repayment is more likely (occurs with probability $(1-\lambda)(1-\alpha)^2$ compared to $(1-\lambda)^2(1-\alpha)^2$) if investments are perfectly correlated. Hence, these three effects dominate the diversification effect that results from investing in different industries, which is why banks prefer to invest in correlated loan portfolios.

In this section we demonstrate that banks always have an incentive to increase the interbank exposure until the government is just able to bail them out. The benefit of being connected to other banks can be further enhanced by choosing correlated assets. This gives banks an incentive to herd. We can thus provide an additional explanation for the herding behavior of banks besides the effect discussed by Acharya and Yorulmazer (2007). In their paper correlated investments increase the bailout probability of each bank. Even if we abstract from the fact that correlated investments increase the bailout

probability, we find an additional incentive for herding behavior. Hence, the mechanism described in this paper leads to an overall increase in systemic risk that results from both interconnectedness as well as herding behavior.

Given that it is optimal for banks to invest in correlated portfolios to maximize their creditors' repayment, we henceforth restrict our analysis to the case where banks invest in correlated investment portfolios.

5. The Interbank Network and Risk Shifting

After showing that it is optimal for banks to invest in correlated investments, we now use this finding and consider the impact of interbank connections on the incentive of banks to engage in risk shifting. To model the riskiness of the investment decision, we consider two assets: a risk-free storage technology that transfers one unit of wealth today into one unit of wealth tomorrow, and a risky negative NPV investment that generates a return $R_N > 1$ with probability $\lambda_N < 1$ such that $\lambda_N R_N < 1$. As in the previous section, banks get c from uninsured creditors and e from equity holders such that $c + e = 1$. Depending on the asset the bank invests in and given that there is no bailout possibility, it can offer creditors either a repayment of c (if it invests in the safe asset) or cR_D^N with probability λ_N and $R_D^N \leq R_N$ if it invests in the risky negative NPV asset. The promised repayment R_D^N results from the binding participation constraint of the equity holder. We assume that the outside option of the equity holder is now given by the risk-free storage technology. Therefore the participation constraint becomes

$$E[d_1] = e \Rightarrow \lambda_N(R_N - cR_D^N) = e \Rightarrow R_D^N = \frac{\lambda_N R_N - (1-c)}{c\lambda_N}$$

Furthermore, since $\lambda_N R_N < 1$

$$R_D^N = \frac{\lambda_N R_N - (1-c)}{c\lambda_N} < \frac{1-(1-c)}{c\lambda_N} = \frac{1}{\lambda_N} \Rightarrow \lambda_N R_D^N < 1$$

We first consider a scenario without a bailout possibility and no interbank network. Here, it can be easily seen that the expected repayment of the creditors is higher if the bank invests in the safe asset since

$$c > \lambda_N c R_D^N \tag{6}$$

Hence, without the possibility of a bailout, banks will always choose the safe investment. Next we consider the case where the bank has a positive probability of being bailed out by the government but still no connections to other banks. Now it can become profitable to switch to the negative NPV investment if the bailout probability is high enough. More precisely, a bank will switch to the negative NPV investment if the expected repayment of creditors for this investment is higher than for the safe repayment c. This yields the condition

$$\lambda_N c R_D^N + (1-\lambda_N)\alpha c R_D^N > c \tag{7}$$

Besides the state of nature where the investment is successful, creditors now also receive the higher return R_D^N when the bank is bailed out by the government. The critical α,

that is, the bailout probability where the bank is indifferent between the two investments is given by

$$\alpha^* = \frac{1 - \lambda_N R_D^N}{(1 - \lambda_N) R_D^N} < 1$$

Hence, for $\alpha > \alpha^*$ it is always profitable to switch to the negative NPV investment. Now we again allow the bank to exchange funds with the bank in the other region. As before, the banks exchange funds K in period 0 in return for a payment of KR_D^N at $t = 1$. Whether banks will switch to the negative NPV investment again depends on α. Whenever the expected repayment of the uninsured creditor from investing in the negative NPV investment opportunity is higher, banks will shift away from the risk-free investment. Formally, the following condition must be satisfied:

$$\lambda_N c R_D^N + (1 - \lambda_N) \left[\alpha c R_D^N + \alpha(1 - \alpha) c R_D^N \frac{K}{c + K} \right] > c \tag{8}$$

Solving this equation for α yields the critical threshold:

$$\alpha^{**} = \frac{c + 2K}{2K} - \sqrt{\frac{(c + 2K)^2}{4K^2} - \frac{(c + K)(R_D^N \lambda_N - 1)}{K R_D^N (\lambda_N - 1)}} \tag{9}$$

We show in the Appendix that the critical α is strictly smaller if a bank is connected (i.e., $K > 0$) to another bank on the interbank market, that is, $\alpha^* > \alpha^{**}$. Hence, the critical threshold α is lower once a bank enters into connections with other banks. Put differently, a lower bailout probability is sufficient to make the bank switch to the negative NPV investment. The positive bailout probability can turn a negative NPV investment into a positive NPV investment from the perspective of the uninsured creditors since they will receive the high repayment with a higher probability. This effect is reinforced once the bank is connected to another bank if this other bank has a positive bailout probability as well. Our results are summarized in the following proposition.

Proposition 5.1. *The more interconnected a bank becomes, the lower the critical bailout probability that makes it profitable for the bank to engage in risk shifting, that is, to switch to negative NPV investments*

Proof See the appendix. QED

Risk shifting thus becomes more attractive for banks since the downside risk is limited by two factors. First, the downside risk is limited by the positive bailout probability because creditors receive their full repayment after the bank is bailed out. Second, the interbank connection further reduces the downside risk, since it adds an additional state where the creditor receives a positive repayment. These two effects turn a negative NPV investment into a positive NPV investment (from the perspective of the uninsured creditors).

Taking the results of Sections 4 and 5 together may help explain why many banks invested in highly correlated low quality assets in the run-up to the financial crisis (e.g., subprime loans). Section 4 shows that interbank connections incentivize banks to invest

in highly correlated portfolios because they benefit from defaulting in states where the banks they are connected to default as well. This section additionally shows that, given that banks prefer correlated investment projects, interbank connections make risk shifting (i.e., investing in low quality assets rather than safe assets) more attractive (as long as there is a positive probability that defaulting banks are bailed out). Hence, one reason for the observed investment behavior prior to the financial crisis may be that the high interconnectedness of large banks incentivized them to invest in highly correlated low quality assets.

6. Extensions

6.1. Three Region Economy

So far we have assumed that the economy consists of only two regions. This gave banks an incentive to increase the funds exchanged, K, in period 0 up to $\overline{K_1}$. Now we want to focus on whether the benefits from taking advantage of the bailout possibility have an influence on the interbank network size and structure. In particular, we analyze the change in the expected utility of the creditors after an additional bank is added to the interbank network. Furthermore, we analyze whether the creditors derive a higher utility if the network is directed or bidirected (see Figure 3).

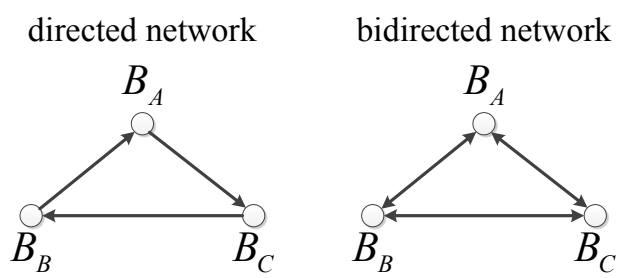

Figure 3: Interbank network structures

Afterwards we investigate how the desired network structure changes if we relax the assumption that the governments in each region (country) can provide exactly the same amount of bailout funds. To derive these results, we extend our model to a three region economy (A, B, and C) and start the analysis by checking whether this improves the expected repayment of the uninsured creditors. First, we examine a directed interbank network. In this case, banks deposit funds K in a neighboring region and receive funds from another neighboring region in return for a payment of KR_D at $t = 1$. Since the model is still symmetric, the expected utility of all uninsured creditors is the same. Hence, it is sufficient to consider only one specific bank and its creditor. In this setup, the expected repayment (U_{DI}) of the uninsured creditors in $t = 1$ becomes

$$U_{DI} = \lambda c R_D + (1-\lambda)\left[\ \alpha c R_D + (1-\alpha)\alpha c R_D \tfrac{K}{c+K} + (1-\alpha)^2 \alpha c R_D \tfrac{K^2}{(c+K)^2}\ \right] \tag{10}$$

To fully capture the respective repayments in the different default states, consider the view of a creditor of bank B_A. If bank B_A is bailed out, the creditor receives the full

repayment. If bank B_A is not bailed out, the repayment of the creditor depends on what happens to the other banks. If bank B_B is bailed out, the creditor receives a fraction $\frac{K}{c+K}$ of his promised repayment. If bank B_B is not rescued but bank B_C is, then the creditor receives a fraction $\frac{K^2}{(c+K)^2}$. Due to the perfect correlation of the banks' investments, the binding participation constraint of the equity holders is again $E[d_1] = \lambda Re$, implying that $R_D = R$. We maintain the assumption of perfect competition, implying that banks must still maximize the expected repayment of their creditors. Hence, the maximization problem for a specific bank becomes

$$\max_K U_{DI} = \lambda cR + (1-\lambda)\left[\ \alpha cR + (1-\alpha)\alpha cR\tfrac{K}{c+K} + (1-\alpha)^2\alpha cR\tfrac{K^2}{(c+K)^2}\ \right] \tag{11}$$

Again, we split the amount of interbank deposits into two intervals. In the interval $K \in [0, \overline{K_1}]$ the government will be able to bail out the bank and repay all liabilities. Hence, for this interval, $\alpha = \alpha_B$ and the derivative of the objective function becomes

$$\frac{\partial U_{DI}}{\partial K} = (1-\lambda)(1-\alpha_B)\alpha_B cR\left[\frac{c}{(c+K)^2} + (1-\alpha_B)\frac{2cK}{(c+K)^3}\right] > 0 \tag{12}$$

Thus, increasing K again enhances the expected utility of the creditor in this interval. If, on the other hand, banks increase their exposure even more, that is, $K \in (\overline{K_1}, \infty]$, the bailout probability α drops to zero. Hence, the expected repayment to C_A drops again to λcR. Thus, in the three region case with a directed interbank network, the expected utility of the uninsured creditors is increasing in K as well, as long as $R(c+K) < \overline{L}$. This implies that banks will choose the same amount of interbank deposits $K = \overline{K_1}$ as in the two region case. Therefore, the highest expected utility that can be achieved is

$$\begin{aligned}
\overline{U_{DI}} &= \lambda cR + (1-\lambda)\left[\begin{array}{l} \alpha_B cR + (1-\alpha_B)\alpha_B cR\frac{\overline{K_1}}{c+\overline{K_1}} \\ +(1-\alpha_B)^2\alpha_B cR\frac{\overline{K_1}^2}{(c+\overline{K_1})^2} \end{array}\right] && (13)\\
&= \lambda cR + (1-\lambda)\left[\begin{array}{l} \alpha_B cR + (1-\alpha_B)\alpha_B \overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2} \\ +(1-\alpha_B)^2\alpha_B \overline{L}\frac{c\overline{K_1}^2}{(c+\overline{K_1})^3} \end{array}\right] && (14)
\end{aligned}$$

Comparing the maximal expected utility of the creditor in a three bank interbank market (see equation (14)) with the two bank case, where the bank in region A is only connected to one other region (see equation (4)), one can easily see that the expected utility increases if the bank is connected to more banks. Since in the three region case each bank is now linked to two other banks (instead of only one other bank) the expected repayment of the uninsured creditors increases. Moreover, the repayment of creditors is again increasing in the interbank exposure K. Therefore, banks will prefer to be connected to two banks instead of only one.

We now consider a bidirected interbank network structure, that is, a structure where each bank has bilateral exposure to all other banks. Since the model is still symmetric, we again restrict our analysis to bank B_A and its creditor. Table 4 summarizes the possible states for this network structure. If the investments are successful, all banks are able to

settle their liabilities and no default occurs (S_1). Hence, the uninsured creditor receives cR_D and the investor receives a dividend $R-R_D$. If the investment fails, the repayment to the uninsured creditors depends on whether the banks are bailed out or not. Several cases must be considered here. If B_A is bailed out by the government (states S_2 to S_4 and state S_6), creditor C_A receives the full repayment. If, however, only one or both of the other banks (B_B and B_C) are rescued, the creditor of bank B_A will receive only a fraction of the contractually specified repayment. In case both other banks are bailed out, each receives an amount $(c+2K)R_D$ from its respective government. Therefore, they are able to fully repay their creditors and settle their interbank claims. Hence, bank B_A receives KR_D from B_B and B_C, respectively, that is, $2KR_D$ in total. Since the bank's total liabilities are $(c+2K)R_D > 2KR_D$, it must split these funds on a pro rata basis among its creditors. Consequently, the uninsured creditor of bank B_A who holds a fraction $\frac{c}{c+2K}$ of the total liabilities receives a total payment of $cR_D\frac{2K}{c+2K}$. The remaining funds are paid back to the other banks.

$\rho = 1$	Prob.	L	B_A	B_B	B_C	C_A	I_A
S_1	λ	S	N	N	N	cR_D	$R - cR_D$
S_2	$(1-\lambda)\alpha^3$	F	B	B	B	cR_D	0
S_3	$(1-\lambda)(1-\alpha)\alpha^2$	F	B	B	N	cR_D	0
S_4	$(1-\lambda)(1-\alpha)\alpha^2$	F	B	N	B	cR_D	0
S_5	$(1-\lambda)(1-\alpha)\alpha^2$	F	N	B	B	$cR_D\frac{2K}{c+2K}$	0
S_6	$(1-\lambda)(1-\alpha)^2\alpha$	F	B	N	N	cR_D	0
S_7	$(1-\lambda)(1-\alpha)^2\alpha$	F	N	B	N	$cR_D\frac{K}{c+K}$	0
S_8	$(1-\lambda)(1-\alpha)^2\alpha$	F	N	N	B	$cR_D\frac{K}{c+K}$	0
S_9	$(1-\lambda)(1-\alpha)^3$	F	N	N	N	0	0

Table 4: Capital flows in a bidirected connected interbank network

We now discuss the states where only one bank receives funds from its government, that is, states S_7 and S_8. The symmetry of our model framework allows us to focus on state S_7, since the cash flows in S_8 can be derived analogously. To derive the exact repayment the uninsured creditor of bank B_A receives, we proceed in several steps. First, we determine the total amount of funds channeled through bank B_A during the repayment process. Since bank B_A is in default and funds are again split on a pro rata basis, the uninsured creditor receives a fraction of $\frac{c}{c+2K}$ of every unit of capital that arrives at bank B_A. The solution strategy is thus as follows: We start by tracking all funds injected into the financial system by the governments and follow these funds until they arrive at bank B_A for the first time. In a next step, we examine the funds that are paid back into the financial system and arrive again at bank B_A. The last step is necessary since capital flows are exchanged continuously between banks B_A and B_C. Note that since bank B_B is bailed out, all its liabilities are settled and hence all funds that arrive at bank B_B stay there.

Next we return to a detailed description of state S_7. In state S_7 only bank B_B is bailed out and thus receives funds of $(c+2K)R_D$, which is sufficient to settle all liabilities, implying that banks B_A and B_C both receive KR_D. A fraction $\frac{K}{c+2K}$ of these funds KR_D that bank B_C receives are passed on to bank B_A. Hence, bank B_A receives an amount $KR_D(1+\frac{K}{c+2K})$ in the first round. As described above, a fraction $\frac{c}{c+2K}$ is directly paid to the uninsured creditor, whereas each of the other banks receives a fraction $\frac{K}{c+2K}$. However,

a fraction of the funds that go to bank B_C flows back to bank B_A. This implies that a fraction $\frac{K^2}{(c+2K)^2}$ is returned to bank B_A after the next cycle flow. After these funds arrive at bank B_A, the same flows occur again. This yields a capital flow to creditor C_A, that can be expressed as a geometric series:

$$\begin{aligned} KR_D\left(1+\frac{K}{c+2K}\right)\sum_{i=0}^{\infty}\left(\frac{K}{c+2K}\right)^{2i}\frac{c}{c+2K} &= KR_D\left(1+\frac{K}{c+2K}\right)\frac{c}{c+2K} \\ &= cR_D\frac{K}{c+K} \end{aligned} \tag{15}$$

As already discussed, state S_8 can be described analogously, implying that the creditor of bank B_A receives the same repayment in this state. Therefore, the expected repayment (U_{BI}) of the uninsured creditors in $t = 1$ can be written as

$$U_{BI} = \lambda cR_D + (1-\lambda)\left[\alpha cR_D + (1-\alpha)\alpha^2 cR_D\frac{2K}{c+2K} + 2(1-\alpha)^2\alpha cR_D\frac{K}{c+K}\right] \tag{16}$$

Again, the participation constraint of the investors implies that $R_D = R$. Due to the fact that U_{BI} is increasing in K until the total liabilities of the bank are equal to $\overline{L}$, the banks will again choose $K = \overline{K_{BI}}$, where $\overline{K_{BI}} = \frac{\overline{L}}{2R_D} - \frac{1}{2}c$ such that $(c+2K)R_D = \overline{L}$. Hence, the maximal expected utility for the uninsured creditor in a bidirected interbank market is

$$\overline{U_{BI}} = \lambda cR + (1-\lambda)\left[\alpha_B cR + (1-\alpha_B)\alpha_B^2 cR\frac{2\overline{K_{BI}}}{c+2\overline{K_{BI}}} + 2(1-\alpha_B)^2\alpha_B cR\frac{\overline{K_{BI}}}{c\overline{K_{BI}}}\right] \tag{17}$$

Now we can compare the highest possible expected utility for creditors in a directed versus a bidirected interbank network. Comparing equations (13) and (17) shows that banks can maximize the expected repayment of their non-insured creditors by trying to establish large directed cycle flows within the interbank market instead of just creating bilateral exposure with other banks. This result can be summarized in the following proposition.

Proposition 6.1. *If all governments can spend equally high amounts for a bailout program, banks in a three region economy are incentivized to create large directed cycle flows instead of bilateral exposures.*

Proof See the Appendix. QED

This result also makes sense intuitively. To make as much use as possible of the bailout possibility, banks prefer being part of a long cycle flow instead of lending money only bilaterally. Thereby, they can benefit to a larger extent from the bailout of any of the banks that are part of the cycle. However, this mechanism only works if a bank can be sure that the other banks will continue to create this large cycle and not start to exchange funds bilaterally.

In a next step, we relax the assumption that the governments in the respective regions can provide the same amount of bailout funds and show how this influences the utility maximizing network structure. Therefore, we assume from now on that there is a different critical threshold $\overline{L}$ for each government (due to different country sizes) where banks become too big to save and therefore the bailout probability decreases to zero. Without loss of generality, we assume that country A can provide more bailout funds than country B, which in turn can provide more than country C. Hence, in the following we assume that $\overline{L_A} > \overline{L_B} > \overline{L_C}$.

In the beginning of this section, we show that the expected repayment of the uninsured creditor is maximized if banks establish a directed interbank network. However, here a directed interbank network is only utility enhancing until bank B_C reaches a balance sheet size of $\overline{L_C}$, which happens at an interbank exposure of K^C where $K_C = \frac{\overline{L_C}}{2R} - \frac{1}{2}c$. Exceeding this threshold would reduce the bailout probability of bank B_C to zero. Hence, if B_C's balance sheet exceeds $\overline{L_C}$, the expected utility for creditor C_C becomes

$$U_{DI}^{C}(K > K^C) = \lambda cR + (1-\lambda)\left[(1-\alpha)\alpha cR\frac{K}{c+K} + (1-\alpha)^2\alpha cR\frac{K^2}{(c+K)^2}\right] \tag{18}$$

Note that this is only true as long as the other two banks are still not too big to save. One can see directly from (18) that the expected repayment of C_C is smaller for $K > K^C$ than for an interbank exposure of $K = K^C$. Therefore, bank B_C does not have an incentive to accept additional funds from other banks as soon as it reaches an interbank exposure of K^C. However, at this point banks B_A and B_B would still be able to increase their interbank exposure to a certain extent without immediately becoming too big to save. Since B_C is not willing to borrow any additional funds on the interbank market, the only option to increase the interbank exposure of B_A and B_B is to lend and borrow bilaterally. Now we must check whether this enhances the expected repayment of creditors C_A and C_B.

Since an additional bilateral interbank exposure between B_A and B_B does not alter the cash flows that are induced by the directed interbank network created by banks B_A, B_B, and B_C, we can consider the bilateral exposure between B_A and B_B in isolation. This added value of bilateral exposure was already discussed in Section 4. Therefore, we can conclude that banks B_A and B_B lend to and borrow from each other until bank B_B becomes too big to save as well. Hence, if governments differ in their ability to bail out banks, banks have an incentive to first establish a connected directed interbank network that includes all banks. As soon as some banks become too big to save they stop their borrowing and lending activities on the interbank market. The remaining banks (which are not yet too big to save) then continue to increase their interbank exposure by establishing directed capital flows between each other. This leads to an interbank network of very high density where the degree centrality of banks is increasing in their size, that is, bigger banks are more connected than smaller banks. Furthermore, our model predicts that larger banks tend to be established in countries with higher bailout possibilities.

Proposition 6.2. *If governments differ in their ability to bail out banks, the density of the interbank network will become very high and the degree centrality of banks will increase in their balance sheet size. Furthermore, large banks will be mainly established in countries with higher bailout possibilities.*

Proof Omitted.

Finally, we relax the assumption that the bailout probability α is not increasing in the interconnectedness of the bank (too interconnected to fail) or in its balance sheet size (too big to fail). As already noted, this reinforces the incentive for banks to have a high interbank exposure K. Furthermore, banks now have the incentive to channel funds through banks that have a very high probability of being bailed out in case of default, which in turn increases the bailout probability of these banks even more. This mechanism may lead to the core bank system that is present in almost all countries and often accounts for the large majority of the total interbank lending.

6.2. Risk Averse Creditors

From now on we allow uninsured creditors to be risk averse (in line with the literature on interbank networks and financial contagion, e.g., Allen and Gale (2000) and Brusco and Castiglionesi (2007)). Here, the interbank market not only is present for the reasons discussed in the previous sections, but also allows banks to co-insure against regional liquidity shocks as in Allen and Gale (2000). We show that even if the interbank market has a different reason to exist, our main mechanism is still present. Specifically, we show that banks have an incentive to increase their interbank exposure beyond the level that would be sufficient to perfectly co-insure against liquidity shocks. Our economy in this section now consists of three dates $t = 0, 1, 2$ and, again, two regions A and B, each with a continuum of identical banks that all adopt the same behavior and can thus be described by a representative bank (protected by limited liability). Furthermore, there are now n ex ante identical uninsured creditors and again one risk-neutral investor. Creditors have Diamond-Dybvig (1983) preferences, that is,

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \omega^i \text{ (early creditors)} \\ u(c_2) \text{ with probability } 1 - \omega^i \text{ (late creditors)} \end{cases}$$

where the utility function $u(\cdot)$ is defined for non-negative numbers, strictly increasing, strictly concave, and twice continuously differentiable and satisfies Inada conditions. Each creditor is endowed with one unit of capital at date $t = 0$. Of the n creditors in each region there are n_e^i early creditors and n_l^i late creditors. Thus $\omega^i \equiv \frac{n_e^i}{n}$ represents the fraction of early creditors, where ω^i can be either high or low ($\omega_H > \omega_L$). There are two equally likely states S_1 and S_2. At date $t = 1$ state-dependent liquidity preferences are revealed (see Table 5).

Each region has the same ex ante probability of facing a high liquidity shock. A creditor's type is private information and the proportion of early creditors in the whole economy is given by $\gamma = \frac{\omega_H + \omega_L}{2}$. Thus, there is no aggregate uncertainty. At $t = 1$ all liquidity-related uncertainty is resolved and creditors learn their type.

There are two types of investment opportunities: a risk-free, liquid type and a risky, illiquid one. The risk-free asset is a storage technology that transfers one unit of capital at a certain period into one unit of capital in the following period. The illiquid asset is only available at date $t = 0$ and generates a return of either $R > 1$ with probability λ or zero with probability $(1 - \lambda)$ at date $t = 2$ for each unit of capital invested. Note that we again assume that the illiquid asset has a positive NPV, that is, $\lambda R > 1$, and that investment outcomes are again perfectly positively correlated across regions.

	A	B
S_1	ω_H	ω_L
S_2	ω_L	ω_H

Table 5: Liquidity shocks

Since our model now has three dates, the equity investors are entitled to receive dividends at $t = 1$ and $t = 2$. Hence, the investor's utility is now

$$u(d_0, d_1, d_2) = \lambda R d_0 + d_1 + d_2$$

As before, since investors can obtain a utility of λRe by immediately consuming the initial endowment, they must earn an expected return of at least λR on their invested money to give up consumption at date $t = 0$. This leads to the following participation constraint for investors:

$$E[d_1 + d_2] \geq e_0 \lambda R$$

Central Planner Economy

In this economy the Pareto-efficient allocation can be characterized as the solution to the problem of a planner maximizing the creditors' expected utility. By pooling resources the planner can overcome the problem of the regions' asymmetric liquidity needs. Let y and x denote the per capita amounts invested in the risk-free and risky assets, respectively. Furthermore, let c and cR_D denote the amounts creditors can withdraw to satisfy their liquidity needs at date $t = 1$ and date $t = 2$, respectively. In this context, R_D can be understood as the interest rate creditors earn by not withdrawing their funds for an additional period. The planner's problem can then be written as

$$\max_{x,y,c,R_D} U = \gamma u(c) + (1 - \gamma) \lambda u(cR_D)$$

subject to

$$x + y \leq n, \ \gamma\, 2nc \leq 2y, \ (1 - \gamma)\, 2ncR_D \leq 2xR,$$
$$x \geq 0, \ y \geq 0, \ c \geq 0, \ R_D \geq 0.$$

The first set of constraints represents budget constraints for periods 0, 1 and 2. Since optimality requires that the constraints be binding, the optimization problem can be rewritten as

$$\max_y \gamma u \left(\frac{y}{\gamma n} \right) + (1 - \gamma) \ \lambda u \left(\frac{R(n - y)}{(1 - \gamma)n} \right)$$

Given the utility function's properties this optimization problem has a unique interior solution. The optimal value $y^* \in (0,1)$ can be obtained from the first-order condition

$$u'\left(\frac{y^*}{\gamma n}\right) = \lambda R u'\left(\frac{R(n-y^*)}{(1-\gamma)n}\right)$$

Once y^* has been determined, we can use the remaining constraints to determine the optimal values of the other variables. Hence, we obtain

$$c^* = \frac{y^*}{\gamma n},\ R_D^* = \frac{R(n-y^*)}{(1-\gamma)nc^*},\ \text{and } x^* = n - y^*$$

Since $\lambda R > 1$, we can conclude that $u'(c) > u'(cR_D)$ and hence $R_D > 1$, implying that consumption is higher at $t = 2$ than at $t = 1$. Consequently, late creditors have no incentive to mimic early creditors. We denote the first-best allocation as $\delta^* = (y^*, x^*, c^*, R_D^*)$.

Decentralized Economy with an Interbank Market and No Bailout Possibility

Allen and Gale (2000) show that this first-best allocation can be achieved by allowing banks in a decentralized economy to co-insure against liquidity shocks. Co-insurance is possible since the liquidity needs of the two regions are negatively correlated. In contrast to Allen and Gale (2000), we again allow banks to exchange an arbitrary amount of deposits K, and not only the amount necessary to achieve first-best. However, we show that exchanging funds above the level of the first best solution does not increase the utility of uninsured creditors if there is no bailout possibility. Let k denote the amount of interbank deposits that is withdrawn by the bank that faces a high liquidity shock at date $t = 1$.

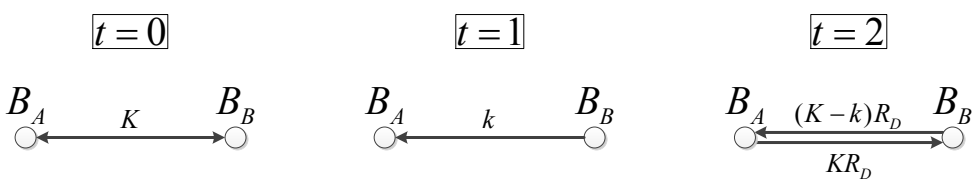

Figure 4: Capital flows in the two region economy

The capital flows are depicted in Figure 4. At $t = 0$ the two banks exchange deposits K. At $t = 1$ the bank with the high liquidity shock (B_A in Figure 4) withdraws an amount k from the other bank to satisfy the liquidity needs of its creditors. In the final period bank B_A receives its remaining deposits $(K-k)$ from bank B_B and pays back the deposits that bank B_B deposited in bank B_A. Additionally, both banks earn a rate of return R_D on these remaining deposits. Furthermore, we assume that contracts again take the form of a standard debt contract, that is, they cannot be made contingent on either the realization of the risky asset or the realization of the state of nature. Hence, each bank can offer a contract $\delta = (y, x, c, R_D, K)$ to its creditors and the bank in the other region. Now R_D additionally represents the gross return paid on interbank deposits held from $t = 1$ until $t = 2$. With perfect competition in the banking sector, the banks will offer their creditors

a contract that replicates the first-best outcome. The optimization problem of a bank can then be written as

$$\max_{x,y,c,R_D,K,k} U = \frac{1}{2}[\omega_H u(c) + (1-\omega_H)\lambda u(cR_D)] + \frac{1}{2}[\omega_L u(c) + (1-\omega_L)\lambda u(cR_D)] \tag{19}$$

subject to

$$\omega_H nc + d_1 \leq y + k \tag{20}$$

$$\omega_L nc + d_1 + k \leq y \tag{21}$$

$$(1-\omega_H)ncR_D + d_2 + KR_D \leq Rx + (K-k)R_D \tag{22}$$

$$(1-\omega_L)ncR_D + d_2 + (K-k)R_D \leq Rx + KR_D \tag{23}$$

$$x \geq 0,\ y \geq 0,\ c \geq 0,\ R_D \geq 0,\ x + y \leq 1 + e_0,\ E[d_1 + d_2] \geq \lambda R e_0,\ k \leq K$$

Constraints (20) and (21) represent budget constraints at date $t = 1$ and constraints (22) and (23) represent budget constraints at date $t = 2$. As shown by Allen and Gale (2000), optimality requires that $k^* = (\omega_H - \gamma)cn$. As long as there is no positive bailout probability, the actual amount of funds exchanged, K, does not alter the utility of the creditors as long as $K \geq k^*$. This leads to the following proposition.

Proposition 6.3. *If there is no possibility for banks to be bailed out and the two representative banks exchange an amount K of deposits, then the first-best allocation δ^* can be implemented by a decentralized banking system offering standard deposit contracts. Moreover, banks have no incentive to exchange more funds than required to achieve first-best, that is, they will only exchange $k^* = (\omega_H - \gamma)cn$.*

Proof For the proof of the first part of the proposition we refer to the proof of Proposition 3 of Brusco and Castiglionesi (2007). To see why the second part holds true, that is, why banks do not exchange more than necessary to achieve first-best, note that optimality again requires the constraints to be binding. Then the amount of funds actually exchanged, K, drops out of the optimization problem. Hence, the amount that is actually exchanged does not influence the utility of the creditors. Therefore, banks have no incentive to exchange more funds than necessary to achieve first-best, which implies that $K = k^* = (\omega_H - \gamma)cn$. QED

This result reconfirms the findings of the previous articles by Allen and Gale (2000) and Brusco and Castiglionesi (2007).

Decentralized Economy with an Interbank Market and Positive Bailout Probability

So far we have assumed that after a bank failure occurs, creditors receive no repayment in period 2. Now we investigate how the results change if there is the possibility that

a bank will be bailed out by the government after a default. As before, we assume this happens with probability α. Therefore, the optimization problem becomes

$$\begin{aligned}\max_{x,y,c,R_D,K,k} U &= \frac{1}{2}\left[\omega_H u(c) + (1-\omega_H)\left[\begin{array}{l}\lambda u(cR_D) + (1-\lambda)[(1-\alpha)^2 u(0) \\ +\alpha(1-\alpha)u(cR_D) \\ +\alpha(1-\alpha)u\left(cR_D\theta_1\right) + \alpha^2 u(cR_D)]\end{array}\right]\right] \\ &+ \frac{1}{2}\left[\omega_L u(c) + (1-\omega_L)\left[\begin{array}{l}\lambda u(cR_D) + (1-\lambda)[(1-\alpha)^2 u(0) \\ +\alpha(1-\alpha)u(cR_D) \\ +\alpha(1-\alpha)u\left(cR_D\theta_2\right) + \alpha^2 u(cR_D)]\end{array}\right]\right] \quad (24)\end{aligned}$$

with

$$\theta_1 = \frac{K-k}{(1-\omega_H)nc + K} \; and \; \theta_2 = \frac{K}{(1-\omega_L)nc + (K-k)}$$

subject to

$$\omega_H nc + d_1 \leq y + k \quad (25)$$

$$\omega_L nc + d_1 + k \leq y \quad (26)$$

$$(1-\omega_H)ncR_D + d_2 + KR_D \leq Rx + (K-k)R_D \quad (27)$$

$$(1-\omega_L)ncR_D + d_2 + (K-k)R_D \leq Rx + KR_D \quad (28)$$

$$x \geq 0,\; y \geq 0,\; c \geq 0,\; R_D \geq 0;\; x + y \leq 1 + e_0;\; E[d_1 + d_2] \geq \lambda R e_0;\; k \leq K$$

Equation (24) is the objective function of the optimization problem of the representative bank in region i. The bank in region i is equally likely to face a high or a low liquidity shock. If a high liquidity shock occurs in, for example, region A, a fraction ω_H of the creditors will withdraw their funds at $t = 1$ and the remaining creditors will demand repayment in $t = 2$. At $t = 2$ several cases must be considered. The risky asset yields a positive return R with probability λ and creditors receive their promised repayment cR_D. If the risky asset yields a zero payoff, the return of the creditor depends on whether the banks are bailed out or not. If neither of the two banks is bailed out, creditors receive no payment. If the bank in region A is bailed out, the government steps in and creditors receive their full repayment cR_D. If only the bank in region B is bailed out, bank B_A receives the funds still owed to it by B_B (see Figure 4). Since B_A has already withdrawn an amount k at date $t = 1$ it receives the remaining funds $(K-k)R_D$. Since B_A has two creditors, namely, its uninsured creditor and bank B_B, funds are again split on a pro rata basis. Hence, creditors receive a fraction θ_1 of their promised repayment. Finally, if both banks are bailed out, then creditors again receive the full amount. The second case (where B_A faces a low liquidity shock) can be described analogously.

All constraints are as in the previous section. By examining the optimization problem, it becomes obvious that the amount of funds exchanged, K, now has an influence on the utility of the creditors. Although K again drops out of the constraints (optimality again requires the constraints to be binding), it now also enters the objective function directly

because it determines the amount that creditors receive in the case of a default if only one bank (here this would be bank B_B) is bailed out. Before the repayment in this state of nature was zero.

Again, optimality requires that banks choose first-best, that is, $k^{**} = (\omega_H - \gamma)cn$. Note, however, that the optimal consumption of creditors (c) changes. Compared to the case without bailout, creditors now consume less in period 1 and increase their consumption in period 2 (we formally show this in the Appendix). This also implies that the optimal amount of funds withdrawn in period 1 is now smaller than in the situation without bailout. Hence, we obtain the following first-order condition for K:

$$\begin{aligned}\frac{\partial U}{\partial K} &= \frac{1}{2}(1-\omega_H)(1-\lambda)\alpha(1-\alpha)c^2 nR_D \frac{(1-\gamma)}{(K+cn(1-\omega_H))^2} u'\left(cR_D \frac{K-cn(\omega_H-\gamma)}{K+cn(1-\omega_H)}\right) \\ &+ \frac{1}{2}(1-\omega_L)(1-\lambda)\alpha(1-\alpha)c^2 nR_D \frac{(1-\gamma)}{(K+cn(1-\gamma))^2} u'\left(cR_D \frac{K}{K+cn(1-\gamma)}\right) \\ &> 0 \end{aligned} \tag{29}$$

As we can see from the first-order condition, the utility of the creditor is now increasing in K (i.e., the funds exchanged in period 0), since K increases the amount that the creditor receives in case of default of the risky asset (although the amount needed to satisfy the consumption needs of creditors is now actually smaller, banks have an incentive to increase their interbank exposure). Therefore, banks have an incentive to increase the amount of interbank deposits and hence their connectivity to a level that exceeds the first-best solution derived before.

Proposition 6.4. *Given a positive bailout probability, banks have an incentive to increase their interbank exposure beyond the first-best level.*

Proof First note that the constraints are the same as in the previous section, where we excluded the possibility of a bailout. Again, optimality requires that the constraints be binding, which implies that K drops out of the constraints. Hence, we only have to examine the objective function. The results follow from the positive derivative of the creditors' utility function with respect to K. QED

Hence, even if the interbank market does not exist only as an insurance for non-insured creditors but also to co-insure against regional liquidity shocks, as in Allen and Gale (2000), the main mechanism is still present. Therefore, banks are still incentivized to increase their interbank exposure as long as they are not too big to save (given that there is a positive bailout probability).

7. Conclusion

This paper sheds light on the puzzle why banks have an incentive to be highly interconnected on the interbank market and why it can be rational to engage in circular lending activities, although this considerably increases systemic risk and leverage without altering the aggregate relation with the real economy. We show that banks create these cyclical liabilities because it enables them to make use of the implicit government

bailout guarantees. Such guarantees shift the probability distribution of the returns of risky investments and thereby increase the expected repayment of uninsured creditors. Furthermore, the mechanism we derive in this paper is able to explain why banks choose correlated investments. Hence, the presented mechanism leads to an overall increase in systemic risk that results from both interconnectedness as well as herding behavior. Moreover, we show that interconnectedness incentivizes banks to engage in risk shifting. Therefore, our model helps explain why banks invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis. Finally, we show that the optimal network structure depends on the amount of funds that is available to bail out banks in different countries. Our results continue to hold even if we allow creditors to be risk averse.

Several policy implications can be derived from our results. Generally, each of these policy implications aims at reducing the banks' incentive to create high interbank exposures by entering into cyclical liabilities. One of the key topics in the current discussion in the European Union is the introduction of a financial transaction tax in order to limit speculative trading activities. Since interconnectedness can not only be created via interbank loans, but also by using derivatives like e.g. CDS, such a tax may be a potential mechanism to reduce the high interconnectedness and therefore mitigate the systemic risk problems that result from investing in highly correlated low-quality assets. Similarly, one can think about increasing the risk weights for interbank loans under the Basel accord and thereby increase the amount of equity necessary to satisfy minimum capital requirements. Currently banks do not have to hold high amounts of capital for most of their interbank exposure. If interbank loans get a higher risk weight, it may incentivize banks to reduce their circular lending activities and hence reduce systemic risk in the interbank market. A third possibility to mitigate the incentives to create large cycle flows would be the introduction of the widely discussed bank levy. Charging banks with large balance sheets (that can very well result from high amounts of cyclical liabilities) higher taxes for their systemic risk can potentially mitigate the incentive to create these large cycle flows in the first place.

Appendix

Switching point K^* in Section 4.2

Here, we will formally derive the critical threshold of interbank deposits K^* (from Section 4.2) that just allows a successful bank to stay solvent if the bank it is connected to defaults and is not bailed out. The critical cases to derive this threshold are those where only one investment fails and neither of the banks is bailed out, i.e. S_8 and S_{11}. Here, the bank with the successful investment will pay the following amount to the bank with the failed investment:

$$\min\left\{KR_D, R\frac{K(c+K)}{c^2+2cK}\right\}$$

The first term represents the amount the successful bank owes to the failed bank and the second term results from:

$$\sum_{i=0}^{\infty} R\left(\frac{K}{c+K}\right)^{(1+2i)} = R\frac{K}{c+K}\frac{1}{1-\frac{K^2}{(c+K)^2}} = R\frac{K(c+K)}{c^2+2cK}$$

Hence, the failing bank receives either its full repayment (if there are enough funds available to settle all claims), i.e. $KR_D \leq R\frac{K(c+K)}{c^2+2cK}$ or receives a payment of $R\frac{K(c+K)}{c^2+2cK}$. The critical threshold up to which the bank receives its full repayment can be written as:

$$K_1^* R_D = R\frac{K_1^*(c+K_1^*)}{c^2+2cK_1^*} \Rightarrow K_1^* = \frac{c(R-cR_D)}{2cR_D-R} \tag{30}$$

From (30) we can see that the successful bank can always pay back its liabilities to the unsuccessful bank as long as $R > 2cR_D$. Thus, it will never default in this case. In what follows we will focus on the more interesting case where a default is possible depending on the level of K. Hence, from now on we will assume that $R < 2cR_D$. We next consider the repayment the uninsured creditor gets from the successful bank. This is given by:

$$\min\left\{cR_D, R\frac{(c+K)}{c+2K}\right\}$$

where the fist term is the total amount owed to the uninsured creditor and the second term comes from:

$$\sum_{i=0}^{\infty} R\frac{c}{c+K}\left(\frac{K}{c+K}\right)^{2i} = R\frac{c}{c+K}\frac{1}{1-\frac{K^2}{(c+K)^2}} = R\frac{(c+K)}{c+2K}$$

Hence, as long as K is small enough such that $cR_D \leq R\frac{(c+K)}{c+2K}$ the successful bank can fully repay its uninsured creditor. However if K exceeds a critical threshold, the bank is unable to settle all its claims and can only repay $R\frac{(c+K)}{c+2K}$ to its creditor. The critical

threshold is given by:

$$cR_D = R\frac{(c+K_2^*)}{c+2K_2^*} \Rightarrow K_2^* = \frac{c(R-cR_D)}{2cR_D - R} \tag{31}$$

As can be seen from (30) and (31), the thresholds K_1^* and K_2^* are the same. We now turn to the repayment of the uninsured creditor of the failed bank. This is given by:

$$\min\left\{cR_D, KR_D\frac{c}{c+K}, R\frac{K}{c+2K}\right\} = \min\left\{cR_D\frac{K}{c+K}, R\frac{K}{c+2K}\right\}$$

where the first term is again the total amount owed to the uninsured creditor, the second term is the maximal payment from the bank with the successful investment to the bank with the failed investment times the fraction the insured creditor gets from this payment, and the last term comes from:

$$\sum_{i=0}^{\infty} R\frac{c}{c+K}\left(\frac{K}{c+K}\right)^{(1+2i)} = R\frac{cK}{(c+K)^2}\frac{1}{1-\frac{K^2}{(c+K)^2}} = R\frac{K}{c+2K}$$

One can immediately see that the unsuccessful bank can never fully repay its uninsured creditors. Furthermore, as long as K is small enough such that $cR_D\frac{K}{c+K} \leq R\frac{K}{c+2K}$, the payment of the unsuccessful bank to its uninsured creditors is $cR_D\frac{K}{c+K}$. If K is too high, the payment is $R\frac{K}{c+2K}$. The critical threshold where this switches is given by

$$cR_D\frac{K_3^*}{c+K_3^*} \leq R\frac{K_3^*}{c+2K_3^*} \Rightarrow K_3^* = \frac{c(R-cR_D)}{2cR_D - R} \tag{32}$$

Hence, all three thresholds are the same, which is why we will denote them in the following by

$$K^* \equiv K_1^* = K_2^* = K_3^*. \tag{33}$$

Therefore, if a specific bank has a successful investment, it is able to settle all its liabilities, even if the other bank fails, as long as its interbank exposure is $K \leq K^*$. This completes the derivation of K^*.

Proof of Corollary 4.2

We now we have to check whether the expected utility for the uninsured creditor is maximized by choosing $K \leq K^*$ or by choosing $K > K^*$. For the interval $K \in [0, K^*]$ we know that:

$$U_0(K \leq K^*) = \lambda cR + (1-\lambda)\alpha cR$$

Therefore, the expected utility of non-insured creditors does not depend on the interbank exposure K. This makes the bank indifferent with regard to the choice of K. For the

interval $K \in [K, \overline{K_0}]$ with $\overline{K_0} = \frac{\overline{L}}{R_D^2} - c$ we know that:

$$\begin{aligned} U_0(K = K^*) &= \lambda cR + (1-\lambda)\alpha cR \\ \frac{\partial U_0}{\partial K}(K^* \leq K \leq \overline{K_0}) &= R\frac{\alpha(1-\alpha)(1-\lambda)c[\lambda + (1-\lambda)\alpha - (1-c)]}{(c+K)^2(\lambda + (1-\lambda)\alpha)} > 0 \end{aligned}$$

Hence, if $(c+K^*)R_D^1 < \overline{L}$, the bank will increase the interbank exposure K until $K = \overline{K_0}$. As soon as this threshold is hit the bailout probability α drops to zero and the expected utility for the uninsured creditors decreases to $\lambda^2 cR_D^2 + \lambda(1-\lambda)R$. If, on the other hand, $(c+K^*)R_D^1 \geq \overline{L}$, the bank will be indifferent about the choice of K in the interval $K = [0, \overline{K_0}]$. Therefore, if $(c+K^*)R_D^1 < \overline{L}$, the bank chooses $K = \overline{K_0}$ in order to maximize the expected utility of its uninsured creditor:

$$\begin{aligned} \overline{U_0} &= \left[\alpha(1+\lambda) + \lambda^2(1-2\alpha) - \alpha^2\lambda(1-\lambda)\right] cR_D^2 \\ &+ \lambda(1-\lambda)(1-\alpha)^2 R + \alpha(1-\lambda)(1-\alpha)cR_D^2 \frac{\overline{K_0}}{c+\overline{K_0}} \end{aligned}$$

In case $(c+K^*)R_D^1 \geq \overline{L}$ the maximal expected utility of its uninsured creditor becomes:

$$\overline{U_0} = \lambda cR + (1-\lambda)\alpha cR$$

This completes the derivation of the expected utility of uninsured creditors in the case of a correlation of zero and the proof of Corollary 4.2.

Proof of Proposition 4.3

To determine whether banks prefer correlated investments, we compare the utility of the uninsured creditors for both types of investment correlations (i.e. a correlation of one and zero) and for the latter case the situations where $(c+K^*)R_D^1 < \overline{L}$ and $(c+K^*)R_D^1 \geq \overline{L}$. First, we consider the case that $(c+K^*)R_D^1 < \overline{L}$:

$$\begin{aligned} \overline{U_1} &> \overline{U_0} \\ \lambda cR + (1-\lambda)\left[\alpha cR + \alpha(1-\alpha)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2}\right] &> \left[\begin{array}{l} \left[\alpha(1+\lambda) + \lambda^2(1-2\alpha) - \alpha^2\lambda(1-\lambda)\right] cR_D^2 \\ +\lambda(1-\lambda)(1-\alpha)^2 R + \alpha(1-\lambda)(1-\alpha)\overline{L}\frac{c\overline{K_0}}{(c+\overline{K_0})^2} \end{array}\right] \end{aligned}$$

After inserting the expression in equation (5) for R_D^2, we can simplify the right hand side and the inequality becomes:

$$\begin{aligned} \lambda cR + (1-\lambda)\left[\alpha cR + \alpha(1-\alpha)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2}\right] &> \left[\begin{array}{l} \frac{\alpha[\alpha-(1-c)]+\alpha\lambda(1-c)(1-\alpha)+\lambda^2 c[1-\alpha(2-\alpha)]}{\alpha+\lambda(1-\alpha)}R \\ +\alpha(1-\lambda)(1-\alpha)\overline{L}\frac{c\overline{K_0}}{(c+\overline{K_0})^2} \end{array}\right] \\ (1-\lambda)\alpha(1-\alpha)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2} &> \alpha(1-\lambda)(1-\alpha)\left[\overline{L}\frac{c\overline{K_0}}{(c+\overline{K_0})^2} - R\frac{(1-c)}{\alpha+\lambda(1-\alpha)}\right] \\ R\frac{1-c}{\alpha+\lambda(1-\alpha)} + \overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2} &> \overline{L}\frac{c\overline{K_0}}{(c+\overline{K_0})^2} \end{aligned}$$

Since the first term on the left hand side is positive and $\frac{cK}{(c+K)^2}$ is decreasing in K as well as $\overline{K_0} > \overline{K_1}$, it follows that $\overline{U_1} > \overline{U_0}$. Next, we consider the case that $(c+K^*)R_D^1 \geq \overline{L}$:

$$\begin{aligned}
\overline{U_1} &> \overline{U_0} \\
\lambda cR + (1-\lambda)\left[\alpha cR + \alpha(1-\alpha)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2}\right] &> \lambda cR + (1-\lambda)\alpha cR \\
(1-\lambda)\alpha(1-\alpha)\overline{L}\frac{c\overline{K_1}}{(c+\overline{K_1})^2} &> 0
\end{aligned}$$

Hence, $\overline{U_1}$ is always larger than $\overline{U_0}$, irrespective of whether $(c+K^*)R_D^1 < \overline{L}$ or $(c+K^*)R_D^1 \geq \overline{L}$. Therefore, the bank always chooses $\rho = 1$. This completes the proof.

Proof of Proposition 5.1

In the following we compare the critical bailout probabilities for the case without (α^*) and with interbank network (α^{**}). By plugging in the critical values derived in Section 5, one can see that:

$$\begin{aligned}
\alpha^* &> \alpha^{**} \\
\frac{1-\lambda_N R_D^N}{(1-\lambda_N)R_D^N} &> \frac{c+2K}{2K} - \sqrt{\frac{(c+2K)^2}{4K^2} - \frac{(c+K)(R_D^N\lambda_N - 1)}{KR_D^N(\lambda_N - 1)}} \\
\sqrt{\frac{(c+2K)^2}{4K^2} - \frac{(c+K)(R_D^N\lambda_N - 1)}{KR_D^N(\lambda_N - 1)}} &> \frac{c+2K}{2K} - \frac{1-\lambda_N R_D^N}{(1-\lambda_N)R_D^N} \\
\frac{(c+2K)^2}{4K^2} - \frac{(c+K)(R_D^N\lambda_N - 1)}{KR_D^N(\lambda_N - 1)} &> \left(\frac{c+2K}{2K} - \frac{1-\lambda_N R_D^N}{(1-\lambda_N)R_D^N}\right)^2 \\
\frac{(c+2K)(1-\lambda_N R_D^N)}{K(1-\lambda_N)R_D^N} - \frac{(c+K)(R_D^N\lambda_N - 1)}{KR_D^N(\lambda_N - 1)} &> \left(\frac{1-\lambda_N R_D^N}{(1-\lambda_N)R_D^N}\right)^2 \\
\frac{K(1-\lambda_N R_D^N)}{K(1-\lambda_N)R_D^N} &> \left(\frac{1-\lambda_N R_D^N}{(1-\lambda_N)R_D^N}\right)^2 \\
R_D^N - \lambda_N R_D^N &> 1 - \lambda_N R_D^N \\
R_D^N &> 1
\end{aligned}$$

This last inequality is always true. This completes the proof.

Proof of Proposition 6.1

In order to show that $\overline{U_{DI}} > \overline{U_{BI}}$ holds, it is sufficient to compare the respective cash flows in case the investments fail, since the success state is equal for both cases. Hence, we have to show that

$$\begin{bmatrix} \alpha_B cR + (1-\alpha_B)\alpha_B cR\frac{\overline{K_1}}{c+\overline{K_1}} \\ +(1-\alpha_B)^2\alpha_B cR\frac{\overline{K_1}^2}{(c+\overline{K_1})^2} \end{bmatrix} > \begin{bmatrix} \alpha_B cR + (1-\alpha_B)\alpha_B^2 cR\frac{2\overline{K_{BI}}}{c+2\overline{K_{BI}}} \\ +2(1-\alpha_B)^2\alpha_B cR\frac{\overline{K_{BI}}}{c\overline{K_{BI}}} \end{bmatrix}$$

After subtracting $\alpha_B cR$ and canceling out $(1-\alpha_B)\alpha_B$ the inequality becomes

$$\frac{\overline{K_1}}{c+\overline{K_1}} + (1-\alpha_B)\frac{\overline{K_1}^2}{(c+\overline{K_1})^2} > \alpha_B\frac{2\overline{K_{BI}}}{c+2\overline{K_{BI}}} + 2(1-\alpha_B)\frac{\overline{K_{BI}}}{c\overline{K_{BI}}}$$

Then we use the information that $\overline{K_{BI}} = \frac{1}{2}\overline{K_1}$ to get

$$\begin{aligned}
\frac{\overline{K_1}}{c+\overline{K_1}} + (1-\alpha_B)\frac{\overline{K_1}^2}{(c+\overline{K_1})^2} &> \alpha_B\frac{\overline{K_1}}{c+\overline{K_1}} + (1-\alpha_B)\frac{\overline{K_1}}{c+\frac{1}{2}\overline{K_1}} \\
\frac{\overline{K_1}^2 c(1-\alpha)}{(c+\overline{K_1})^2(2c+\overline{K_1})} &> 0
\end{aligned}$$

Since in the last line all terms on the left hand side are always positive, it holds that $\overline{U_{DI}} > \overline{U_{BI}}$. This completes the proof.

Discussion of optimal consumption with risk-averse creditors and positive bailout probability

To understand why the optimal consumption decreases in $t = 1$ if a bailout is possible first note that a bailout simply changes the probability distribution of the investment. Without bailout creditors receive funds for consumption only with probability λ in period 2. Now if the investment fails there is still a positive probability that creditors receive (at least parts of) their funds. To fully capture the optimal consumption decision we look at a situation where the investment returns and respective probabilities match exactly those of the risky investment considered in the paper when there is a positive bailout probability.

$$\max_{x,y,c,R_D} U = \gamma u(c) + (1-\gamma)\left[\lambda u(cR_D) + (1-\lambda)[\alpha u(cR_D) + \alpha(1-\alpha)u(\theta cR_D)]\right] \tag{34}$$

subject to

$$x + y \leq n,\ \gamma\, 2nc \leq 2y,\ (1-\gamma)\, 2ncR_D \leq 2xR,$$
$$x \geq 0,\ y \geq 0,\ c \geq 0,\ R_D \geq 0.$$

Since the constraints in the respective periods again have to be binding we can solve them for c and R_D, respectively and can plug these values into the objective function. This yields:

$$\max_y U = \gamma u\left(\frac{y}{\gamma n}\right) + (1-\gamma)\left[\begin{array}{l} \lambda u\left(\frac{R(n-y)}{(1-\gamma)n}\right) \\ +(1-\lambda)\left[\alpha u\left(\frac{R(n-y)}{(1-\gamma)n}\right) + \alpha(1-\alpha)u\left(\theta\frac{R(n-y)}{(1-\gamma)n}\right)\right] \end{array}\right]$$

The first order condition with respect to y then yields:

$$u'\left(\frac{y}{\gamma n}\right) = u'\left(\frac{R(n-y)}{(1-\gamma)n}\right)[\lambda R + (1-\lambda\alpha R)] + (1-\lambda)\theta R\alpha u\left(\theta\frac{R(n-y)}{(1-\gamma)n}\right) \tag{35}$$

Looking at this first order condition one can see that the marginal utility of consumption in period 1 is higher now, implying that consumption is lower. Hence, if it is more likely to get the higher repayment at $t = 2$ creditors want to shift more consumption to this later period. This completes the discussion of the optimal consumption allocation with risk-averse creditors and positive bailout probability.

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